PHYSICAL REVIEW C, VOLUME 60, 051305

Simple model for decay of superdeformed nuclei

C. A. Stafford and B. R. Barrett
Physics Department, University of Arizona, 1118 East 4th Street, Tucson, Arizona 85721

(Received 28 May 1999; published 7 October 1999)

Recent theoretical investigations of the decay mechanism out of a superdeformed nuclear band have yielded qualitatively different results, depending on the relative values of the relevant decay widths. We present a simple two-level model for the dynamics of the tunneling between the superdeformed and normal-deformed bands, which treats decay and tunneling processes on an equal footing. The previous theoretical results are shown to correspond to coherent and incoherent limits of the full tunneling dynamics. Our model accounts for experimental data in both the $A\sim150$ mass region, where the tunneling dynamics are coherent, and in the $A\sim190$ mass region, where the tunneling dynamics are incoherent. [S0556-2813(99)50911-4]

PACS number(s): 21.60.−n, 21.10.Re, 27.70.+q, 27.80.+w

One of the most interesting recent discoveries in nuclear-structure physics is the existence of superdeformation for nuclei in the mass $A\sim150$ and $A\sim190$ regions. So far, a consistent theory regarding the decay out of a superdeformed (SD) rotational band into a normal-deformed (ND) band has not been achieved. Most of the early work [1–6] on this problem attributed the decay-out process to a mixing of the SD states with ND states of equal spin. Decay out of the SD band sets in at a spin I_0, for which penetration through the barrier between the SD minimum and the ND minimum is competitive with the E2 decay within the SD band. A statistical model was used [2,3] to describe the ND states, and the decay out of the SD band was determined as a function of the decay widths Γ_S and Γ_N in the SD and ND potential wells, respectively, the spreading (or tunneling) width $\Gamma^{\downarrow}$ through the barrier, and the average spacing D_N of the ND states. Under the assumption that the ND states form a continuum on the scale of the other energies in the problem, the spreading width was found, using Fermi's golden rule, to be [5]

$$\Gamma^{\downarrow}=2\pi\langle V^2\rangle/D_N, \tag{1}$$

where $\langle V^2\rangle$ is the mean square of the coupling matrix elements $V_{\alpha\beta}$ connecting the SD and ND states. $\Gamma^{\downarrow}$ measures the strength of the coupling between the SD and ND states. In Refs. [2,3], it was assumed that $\Gamma_N\lesssim\Gamma_S$ and that $\Gamma^{\downarrow}/D_N\sim1$, i.e., that the coupling between the SD and ND states is relatively strong.

Quite recently, a different approach to this problem has been reported [7]. In this approach, the reduction factor F_S of the intraband transition intensity is calculated directly as a function of the spreading width $\Gamma^{\downarrow}$ and of the intraband E2 width Γ_S. Their final result for F_S is shown to be independent of the statistical E1 decay widths Γ_N of the ND states, provided that $\Gamma_N\gg\Gamma^{\downarrow},\Gamma_S$. Since the publication of the later result, it has been difficult to reconcile the predictions of these two calculations, because their final results do not depend upon the same parameters.

The purpose of the present Rapid Communication is to formulate a simple two-level model for this problem, so as to study in detail the dependence of the decay-out process on Γ_S, Γ_N, and $\Gamma^{\downarrow}$. It will be shown that the results of both previous investigations of the decay-out process can be obtained in certain limits, depending on the relative sizes of these widths. It will also be shown that the appropriate expression for the spreading width is not Eq. (1), but

$$\Gamma^{\downarrow}=\frac{2\overline{\Gamma}V^2}{\Delta^2+\overline{\Gamma}^2}, \tag{2}$$

where $\overline{\Gamma}=(\Gamma_S+\Gamma_N)/2$, V is the matrix element connecting the SD state of interest with the single ND state with which it mixes most strongly, and Δ is the energy difference of these two states. It will be shown that Eq. (1) is indeed the correct mean value of $\Gamma^{\downarrow}$ over a statistical ensemble of nuclei in the limit $V\ll\overline{\Gamma}$, in agreement with Ref. [7]. However, the fluctuations in $\Gamma^{\downarrow}$ are typically much larger than its mean, indicating that Eq. (2) must be used to describe the decay out of a particular superdeformed state.

Motivated by the experimental fact [8] that $\Gamma_N,\Gamma_S\ll D_N$ in the $A\sim190$ region, we consider an effective two-level system consisting of the superdeformed state S and the normal-deformed state N to which it couples most strongly. The Hamiltonian of the system is

$$H=H_0+H_D, \tag{3}$$

where H_D describes the electromagnetic decay processes Γ_N and Γ_S *within* the ND and SD bands, respectively, and

$$H_0=\varepsilon_S c_S^{\dagger}c_S+\varepsilon_N c_N^{\dagger}c_N+V(c_S^{\dagger}c_N+c_N^{\dagger}c_S) \tag{4}$$

describes the effective two-level system, including tunneling through the barrier separating the SD and ND states. Here $c_S^{\dagger}$ and $c_N^{\dagger}$ are creation operators for the superdeformed state S, of energy ε_S, and the normal-deformed state N, of energy ε_N, respectively. Without loss of generality, the tunneling matrix element V is chosen positive via an appropriate choice of the relative phase of the states S and N.

In order to include both the coherent "Rabi oscillations" due to V and the irreversible decays Γ_S and Γ_N, it is useful to consider the retarded Green's function

$$G_{ij}(t)=-i\theta(t)\langle\{c_i(t),c_j^{\dagger}(0)\}\rangle \tag{5}$$

and its Fourier transform

$$G_{ij}(E)=\int_{-\infty}^{\infty} dt G_{ij}(t)e^{iEt}. \tag{6}$$

The Green's function of the tunneling Hamiltonian H_0 satisfies

$$G_0^{-1}(E)=\mathbf{1}(E+i0^+)-H_0, \tag{7}$$

where $\mathbf{1}$ is the unit matrix. In the $|S\rangle,|N\rangle$ basis, one has

$$G_0^{-1}(E)=\begin{pmatrix} E-\varepsilon_S+i0^+ & -V \\ -V & E-\varepsilon_N+i0^+ \end{pmatrix}. \tag{8}$$

The full Green's function, including decay processes, may be calculated from Dyson's equation,

$$G^{-1}=G_0^{-1}-\Sigma, \tag{9}$$

where Σ is the self-energy matrix describing the decay processes Γ_S and Γ_N induced by H_D. The simplest ansatz for Σ is [7]

$$\Sigma\equiv\begin{pmatrix} \Sigma_{SS} & \Sigma_{SN} \\ \Sigma_{NS} & \Sigma_{NN} \end{pmatrix}=\begin{pmatrix} -i\Gamma_S/2 & 0 \\ 0 & -i\Gamma_N/2 \end{pmatrix}. \tag{10}$$

Using Eqs. (8) and (10), one can solve Dyson's equation to obtain the full retarded Green's function of the two-level system,

$$\begin{aligned} G&\equiv\begin{pmatrix} G_{SS} & G_{SN} \\ G_{NS} & G_{NN} \end{pmatrix} \\ &=[(E-\varepsilon_S+i\Gamma_S/2)(E-\varepsilon_N+i\Gamma_N/2)-V^2]^{-1} \\ &\quad\times\begin{pmatrix} E-\varepsilon_N+i\Gamma_N/2 & V \\ V & E-\varepsilon_S+i\Gamma_S/2 \end{pmatrix}. \end{aligned} \tag{11}$$

One can obtain from G all information about the dynamics of the system and the branching ratios of the decay processes.

Let us first study the dynamics of the coupled SD-ND system. Assuming the nucleus starts out at time zero in the superdeformed state $|S\rangle$, the probability that the nucleus is in state $|S\rangle$ at a later time t is $P_S(t)=|G_{SS}(t)|^2$. The probability that the nucleus is in the normal state $|N\rangle$ at time t is $P_N(t)=|G_{NS}(t)|^2$. $P_S(t)$ and $P_N(t)$ may be calculated straightforwardly from the Fourier transform of Eq. (11). The general result for $P_N(t)$ is

$$P_N(t)=\frac{2V^2}{|\omega|^2}e^{-\bar{\Gamma}t}(\cosh\omega_i t-\cos\omega_r t), \tag{12}$$

where

$$\omega\equiv\omega_r+i\omega_i=\sqrt{4V^2+(\Delta-i\Gamma')^2}, \tag{13}$$

$\Delta=\varepsilon_N-\varepsilon_S$, $\Gamma'=(\Gamma_N-\Gamma_S)/2$, and $\bar{\Gamma}$ was defined after Eq. (2). The general expression for $P_S(t)$ is rather lengthy.

The tunneling dynamics are particularly interesting when the energy difference $\Delta=0$. There are then two qualitatively different dynamical regimes, depending on the relative size of the tunneling matrix element V and the difference in decay rates Γ'. For $2V>|\Gamma'|$, the tunneling dynamics are *coherent*, and one finds

$$P_S(t)=e^{-\bar{\Gamma}t}\left[\cos^2\frac{\omega_0 t}{2}+\frac{\Gamma'}{\omega_0}\sin\omega_0 t+\frac{\Gamma'^2}{\omega_0^2}\sin^2\frac{\omega_0 t}{2}\right],$$

$$P_N(t)=\frac{4V^2}{\omega_0^2}e^{-\bar{\Gamma}t}\sin^2\frac{\omega_0 t}{2}, \tag{14}$$

where the Rabi frequency ω_0 is

$$\omega_0=|4V^2-\Gamma'^2|^{1/2}. \tag{15}$$

For $2V<|\Gamma'|$, on the other hand, the tunneling dynamics are *incoherent*, and

$$P_S(t)=e^{-\bar{\Gamma}t}\left[\cosh^2\frac{\omega_0 t}{2}+\frac{\Gamma'}{\omega_0}\sinh\omega_0 t+\frac{\Gamma'^2}{\omega_0^2}\sinh^2\frac{\omega_0 t}{2}\right],$$

$$P_N(t)=\frac{4V^2}{\omega_0^2}e^{-\bar{\Gamma}t}\sinh^2\frac{\omega_0 t}{2}. \tag{16}$$

For $\Gamma'=0$ and $\Delta\neq 0$, the tunneling dynamics are *coherent*, given by Eq. (14), with $\omega_0\to(4V^2+\Delta^2)^{1/2}$. For $\Gamma'\neq 0$ and $\Delta\neq 0$, the tunneling dynamics have both coherent and incoherent components [c.f. Eq. (12)], the coherent component being suppressed for large Γ' and/or large Δ.

The dynamics of the system are similar to that of the two-level system with dissipation, investigated by Leggett *et al.* [9]; the principal difference is that we consider a two-level system in which the total number of particles is itself time dependent. The physical origin of the imaginary self-energy Σ is virtual transitions of the nucleus to lower-lying states and back again, which alter the state of the electromagnetic environment. This is analogous to the coupling of the two-level system to a bath of bosonic excitations considered in Ref. [9]. If the environment couples with equal strength to the states S and N, i.e., if $\Gamma_S=\Gamma_N=\bar{\Gamma}$, the Green's function factorizes quite generally [10]; $G(t)=e^{-\bar{\Gamma}t/2}G_0(t)$, and the nucleus undergoes Rabi oscillations with frequency $\omega=(4V^2+\Delta^2)^{1/2}$ between the states S and N. The nucleus is in a coherent superposition of states, which decays at an overall rate $\bar{\Gamma}$ to lower-lying states. However, if the environment couples with different strengths to the states S and N, i.e., if $\Gamma'\neq 0$, coherent tunneling between S and N is suppressed since the environment ''measures'' which state the system is in. For $\Delta=0$ and $0<|\Gamma'|<2V$, the dynamics described by Eq. (14) are qualitatively similar to the case $\Gamma'=0$, but the Rabi frequency is reduced to the value given in Eq. (15). If the difference in coupling exceeds the critical value $|\Gamma'|>2V$ for $\Delta=0$, the coherent superposition of S and N is destroyed altogether, and the dynamics are overdamped. As in the model of Ref. [9], there are both coherent and incoherent components of the dynamics when both $\Gamma' \neq 0$ and $\Delta \neq 0$.

Let us now turn our attention to the decay branching ratio, which is the experimentally measurable quantity. When the nucleus is in the state *S*, it decays at a rate Γ_S to a lower superdeformed state, and when it is in state *N* it decays at a rate Γ_N to a lower-energy state in the normal-deformed band. Thus, the time-dependent rates to decay in the *S* and *N* channels are

$$\tilde{\Gamma}_S(t) = \Gamma_S P_S(t), \ \tilde{\Gamma}_N(t) = \Gamma_N P_N(t). \tag{17}$$

The fraction F_N of nuclei that decay via *E*1 processes in the normal-deformed band is just [11]

$$F_N = \frac{\int_0^\infty dt \tilde{\Gamma}_N(t)}{\int_0^\infty dt [\tilde{\Gamma}_N(t) + \tilde{\Gamma}_S(t)]} = \Gamma_N \int_0^\infty dt P_N(t). \tag{18}$$

This integral may be evaluated to obtain the central result of this paper,

$$F_N = \frac{(1+\Gamma_N/\Gamma_S)V^2}{\Delta^2 + \bar{\Gamma}^2(1+4V^2/\Gamma_N\Gamma_S)}. \tag{19}$$

The fraction of nuclei decaying via *E*2 processes within the superdeformed band is $F_S = 1 - F_N$. In Ref. [7], F_S was denoted by *F*.

Let us now consider some limits of Eq. (19). In the limit of very strong coupling of the states *S* and $N, V \gg \bar{\Gamma}$, one finds

$$\lim_{V/\bar{\Gamma} \to \infty} F_N = \frac{\Gamma_N(\Gamma_S + \Gamma_N)}{(\Gamma_S + \Gamma_N)^2 + \Gamma_S \Gamma_N (\Delta/V)^2}. \tag{20}$$

This is equivalent to the result of Vigezzi, Broglia, and Dossing [2,3] for the case where only a single SD state and a single ND state mix:

$$F_N^{vbd} = \sum_{\sigma = \pm} \frac{|c_\sigma|^2 (1 - |c_\sigma|^2) \Gamma_N}{(1 - |c_\sigma|^2)\Gamma_N + |c_\sigma|^2 \Gamma_S}, \tag{21}$$

where $c_\pm = \langle \pm | S \rangle$ are the mixing amplitudes of the eigenstates $|\pm\rangle$ of H_0 with $|S\rangle$. From Eq. (4) we have

$$|c_\pm|^2 = [1 + (x \pm \sqrt{x^2+1})^2]^{-1}, \tag{22}$$

with $x = \Delta/2V$. Inserting Eq. (22) into Eq. (21), one indeed recovers Eq. (20). Thus the result (21) of Refs. [2,3] is seen to be a limiting case for $V/\bar{\Gamma} \to \infty$ (i.e., for *fully coherent* tunneling) of our general result, Eq. (19).

Another limit was considered in Ref. [7], namely $\Gamma_N \gg \Gamma_S, \Gamma^\downarrow$. In this limit, the tunneling dynamics are *incoherent*. The assumption $\Gamma_S \ll \Gamma_N$ is motivated by the fact that Γ_S is an *E*2 decay and Γ_N is an *E*1 decay. In the limit $\Gamma_N \gg \Gamma_S$, Eq. (19) simplifies to

$$\lim_{\Gamma_N/\Gamma_S \to \infty} F_N = \frac{\Gamma^\downarrow}{\Gamma_S + \Gamma^\downarrow}, \tag{23}$$

where $\Gamma^\downarrow$ is given by Eq. (2). Equation (23) is identical to the principal result of Weidenmüller, von Brentano, and Barrett [7], although our expression for $\Gamma^\downarrow$ differs from that of Ref. [7]. Note that, in contrast to the argument of Ref. [7], no assumption has been made about the relative size of *V* and $\bar{\Gamma}$ in deriving Eq. (23) from Eq. (19). However, the interpretation of $\Gamma^\downarrow$ as a *tunneling rate* is only justified when $\Gamma^\downarrow/\bar{\Gamma} \ll 1$; for larger values of *V*, the tunneling dynamics described by Eq. (12) are more complex, though the integrated rate still obeys Eq. (23), provided $\Gamma_N \gg \Gamma_S$.

Equation (2) for $\Gamma^\downarrow$ is also the expression one would obtain from a correct application of Fermi's golden rule in the limit $V \ll \bar{\Gamma} \ll D_N$:

$$\Gamma^\downarrow = 2\pi V^2 \int_{-\infty}^{\infty} dE \rho_S(E) \rho_N(E), \tag{24}$$

where the lifetime-broadened densities of states of the SD and ND levels are

$$\rho_S(E) = \frac{\Gamma_S/2\pi}{(E-\varepsilon_S)^2 + \Gamma_S^2/4},$$

$$\rho_N(E) = \frac{\Gamma_N/2\pi}{(E-\varepsilon_N)^2 + \Gamma_N^2/4}.$$

Evaluating the integral in Eq. (24), one obtains expression (2). The level-spacing D_N in the ND band is irrelevant if $V \ll D_N$, since *V* only mixes the state *S* and the single state *N* which is closest to it in energy in that case, as we have assumed.

The branching ratio (19) depends strongly on the energy difference $\Delta = \varepsilon_N - \varepsilon_S$, which in turn depends sensitively on the microscopic Hamiltonian of the particular nucleus under investigation. In order to eliminate this parameter dependence, one practice that is employed is to calculate the average of F_N over a statistical Gaussian orthogonal ensemble of similar nuclei. In the limit of incoherent tunneling $V \ll \Gamma_S, \Gamma_N$, Eq. (19) may be integrated over Δ to obtain

$$\langle F_N \rangle = \int_{-\infty}^{\infty} \frac{d\Delta}{D_N} F_N(\Delta) = \frac{\langle \Gamma^\downarrow \rangle}{\Gamma_S}, \tag{25}$$

where $\langle \Gamma^\downarrow \rangle$ is given by the right-hand side of Eq. (1), in agreement with Ref. [7]. However, it is clear from Eq. (19) that F_N typically deviates significantly from its mean value. For instance, the mean square of F_N is much larger than the square of $\langle F_N \rangle$ when $D_N \gg \bar{\Gamma}$:

$$\frac{\langle F_N^2 \rangle}{\langle F_N \rangle^2} = \frac{D_N \langle V^4 \rangle}{2\pi \bar{\Gamma} \langle V^2 \rangle^2}. \tag{26}$$

TABLE I. Widths and level spacings for a number of nuclei, deduced from the data of Refs. [2,8,12], following the analysis of Ref. [2] for $A\sim150$ and of Ref. [7] for $A\sim190$. The spin values of the decaying states are given in parentheses.

Nucleus	Γ_N (meV)	D_N (eV)	Γ_S (meV)	$\Gamma^{\downarrow}$ (meV)
^{152}Dy(26)	10–20	3–10	$\sim$11	900–3000
^{152}Dy(24)	10–20	3–10	$\sim$7.6	900–3000
^{192}Hg(12)	10.3	34	0.116	0.018
^{192}Hg(10)	10.3	30	0.054	0.544
^{194}Hg(12)	18.1	92	0.144	0.097
^{194}Hg(10)	18.4	79	$\geqslant$0.047	$\geqslant$0.89
^{194}Pb(10)	1.6	1699	0.066	0.011
^{194}Pb(8)	1.7	1549	0.028	0.009

Thus, it would be preferable to compare experimental results directly with Eq. (19), rather than with its ensemble average.

In Table I, we show some experimental data for nuclei in the $A\sim150$ and $A\sim190$ mass regions. So far, little data is available in the $A\sim150$ region, with estimates only [2] for the widths for ^{152}Dy. From the numbers given in Table I, we observe that the nuclei in the $A\sim190$ mass region have $\Gamma^{\downarrow},\Gamma_S \ll \Gamma_N$. The dynamics of $S\rightarrow N$ tunneling in these nuclei are thus incoherent, and the appropriate branching ratio is given by Eq. (23), in agreement with Ref. [7]. On the other hand, for ^{152}Dy, $\Gamma^{\downarrow}\gg\Gamma_S,\Gamma_N$, indicating coherent $S\rightleftharpoons N$ tunneling. The measured value [2,12] of $F_S=0.51$ for ^{152}Dy (26) is consistent with Eq. (20), using the values of Γ_S and Γ_N in Table I and assuming $V/\Delta\sim1$, in accordance with the theory of Refs. [1–6]. Our general result (19) is consistent with the data in both the $A\sim150$ and $A\sim190$ mass regions and unifies these two complementary theoretical approaches.

A recent paper [13] shows that the $S\rightarrow N$ tunneling rate may be enhanced by several orders of magnitude if the ND states are chaotic at the moment of the decay out. These results are not inconsistent with our two-level model, but would simply imply an enhancement of the tunneling matrix element V.

In conclusion, we have shown that a simple two-level model, which treats decay and tunneling processes on an equal footing, can explain the apparently disparate previous theoretical results, i.e., Refs. [1–6] versus Ref. [7], for the decay out of a superdeformed band. These previous results are shown to correspond to the coherent and incoherent limits, respectively, of the tunneling dynamics, and are special cases of our general result, Eq. (19). We remark that it is straightforward to extend our method to treat an SD state coupled to an arbitrary number of ND states.

Note added in proof. In a recent paper [14], Gu and Weidenmüller have demonstrated that the result of Ref. [7] has a large variance in the $A\sim190$ mass region, consistent with our Eq. (26) for the ensemble average. Nonetheless, we have shown that for $A\sim190$, our Eq. (23) holds for each member of the ensemble. The large variance of F_N should, thus, be attributed to the fluctuations of $\Gamma^{\downarrow}$ over the ensemble [see Eq. (2)].

B.R.B. thanks R. Krücken for helpful discussions and acknowledges partial support from NSF Grant No. PHY9605192.

[1] K. Schiffer, B. Herskind, and J. Gascon, Z. Phys. A **332**, 17 (1989).
[2] E. Vigezzi, R. A. Broglia, and T. Dossing, Phys. Lett. B **249**, 163 (1990).
[3] E. Vigezzi, R. A. Broglia, and T. Dossing, Nucl. Phys. **A520**, 179c (1990).
[4] Y. R. Shimizu, F. Barranco, R. A. Broglia, T. Dossing, and E. Vigezzi, Phys. Lett. B **274**, 253 (1992).
[5] Y. R. Shimizu, E. Vigezzi, T. Dossing, and R. A. Broglia, Nucl. Phys. **A557**, 99c (1993).
[6] T. L. Khoo *et al.*, Nucl. Phys. **A557**, 83c (1993).
[7] H. A. Weidenmüller, P. von Brentano, and B. R. Barrett, Phys. Rev. Lett. **81**, 3603 (1998).
[8] R. Kühn, A. Dewald, R. Krücken, C. Meier, R. Peusquens, H. Tiesler, O. Vogel, S. Kaseman, P. von Brentano, D. Bazzacco, C. Rossi-Alvarez, S. Lunardi, and J. de Boer, Phys. Rev. C **55**, R1002 (1997).
[9] A. J. Leggett, S. Chakravarty, A. T. Dorsey, M. P. A. Fisher, A. Garg, and W. Zwerger, Rev. Mod. Phys. **59**, 1 (1987).
[10] C. A. Stafford and N. S. Wingreen, Phys. Rev. Lett. **76**, 1916 (1996).
[11] It should be mentioned that the branching ratios can also be expressed directly in terms of the energy Green's functions using Parseval's theorem:
$$F_N=\Gamma_N\int_{-\infty}^{\infty}\frac{dE}{2\pi}|G_{NS}(E)|^2,\quad F_S=\Gamma_S\int_{-\infty}^{\infty}\frac{dE}{2\pi}|G_{SS}(E)|^2.$$
[12] B. Singh, R. B. Firestone, and S. Y. Frank Chu, *Table of Superdeformed Nuclear Bands and Fission Isomers*, 2nd Ed. (LBL, Berkeley, 1996), pp. 8, 84–86.
[13] S. Åberg, Phys. Rev. Lett. **82**, 299 (1999).
[14] J.-Z. Gu and H. A. Weidenmüller, MPIK preprint, Heidelberg, 1999.