Quantum-Dot Cascade Laser: Proposal for an Ultralow-Threshold Semiconductor Laser

Ned S. Wingreen and Charles A. Stafford

***Abstract*—We propose a quantum-dot version of the quantum-well cascade laser of Faist *et al.* The elimination of single phonon decays by the three-dimensional confinement implies a several order-of-magnitude reduction in the threshold current. The requirements on dot size (10–20 nm) and on dot density and uniformity [one coupled pair of dots per (200 nm^3) with 5% nonuniformity] are close to current technology.**

***Index Terms*— CW lasers, proposals, quantum-well lasers, semiconductor lasers, submillimeter wave lasers.**

The RECENT demonstration by Faist *et al.* [1] of a laser based on a cascade of coupled quantum wells (QW's) has opened up new possibilities in semiconductor lasers. Here we explore one possibility aimed at reducing the threshold current: a version of the Faist *et al.* laser based on quantum dots rather than QW's. While there have been various proposals for low-threshold quantum-dot lasers [2]–[4], all have been based on electron-hole recombination. Recent work offers promise of realizing a quantum-dot laser of the electron-hole type [5]–[7], but problems remain, including the slow energy relaxation of electrons in the dots which leads to poor recombination efficiency [8].

The quantum-dot cascade laser we propose here offers the advantages of an intrinsically strong and narrow gain spectrum, with a minimal rate of nonradiative decays. As in the quantum-well (QW) cascade laser, the current directly pumps the upper lasing level so there is no problem of slow relaxation [8]. However, unlike the QW cascade laser, nonradiative decay by phonon emission can be eliminated. Since the nonradiative rate of decay due to phonon emission in QW's is 3000 times the radiative decay rate [1], elimination of phonon decays is a priority. To eliminate phonon emission in the proposed quantum-dot scheme requires dots smaller than 10–20 nm in all three dimensions. This is the primary technological difficulty, but there is reason to believe that such dimensions can be achieved [9]–[11].

In what follows, we will describe the proposed quantum-dot laser in more detail and compare it to the QW cascade laser [1]. The dot size requirements will be estimated as well as the dot density and uniformity requirements. The latter follow from a comparison of the gain coefficient to the typical effective loss in semiconductor injection lasers. Finally, the threshold current for lasing is estimated from the total rate of spontaneous emission.

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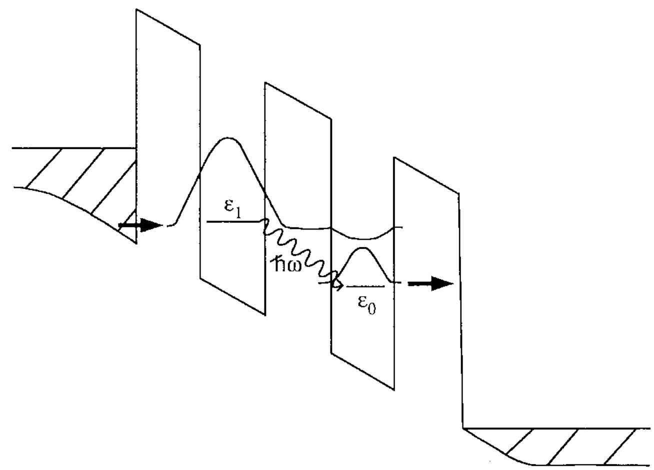

Fig. 1. Schematic conduction-band energy diagram of active region of proposed quantum-dot cascade laser. For low-threshold-current operation, the energy difference between the first excited state and the ground state of the coupled dots, $\epsilon_1 - \epsilon_0$, must be larger than all phonon energies.

It is profitable to compare and contrast the proposed quantum-dot laser and the QW laser of Faist *et al.* [1] using the simplified conduction-band energy diagram in Fig. 1, which suffices for both. The diagram shows two electronically coupled dots or wells [12].[1] Photons are generated by the transition of an electron from the first excited state to the ground state of the coupled dots or coupled wells. In both cases, electrons are injected directly into the excited state by a current tunneling through the upstream barrier. Once an electron is de-excited, it escapes quickly through the downstream barrier, so that photon absorption is negligible.

The essential difference between the quantum-dot and QW lasers is that in the former the excited and ground-state electronic levels, shown in Fig. 1, represent truly discrete states, while in the latter each represents the bottom of a continuous band of states. Specifically, in the QW case, electrons form bands due to their free movement in the two dimensions transverse to the direction of conduction-band energy variation shown in Fig. 1 (i.e., the direction of current flow). As a consequence, the dominant electronic decay mechanism in the QW's is nonradiative, involving emission of an optic phonon rather than a photon. Since the bands are continuous in energy, such transitions are always allowed, and since the electron-optic-phonon coupling is much stronger than

[1] In practice, QW cascade lasers contain three wells and/or a superlattice Bragg reflector to control the rate of tunneling escape.

the electron-photon coupling, such nonradiative transitions will always dominate the radiative ones.

In contrast, in the quantum-dot laser, the rate of radiative decay may dominate the nonradiative rate. Since the excited and ground states of the coupled dots are discrete levels, nonradiative decays will involve emission of a phonon at the difference energy. In general, phonon energies form a continuous band so that such one-phonon decays are allowed. However, if the difference energy is larger than the largest phonon energy (e.g., the optic phonon energy at $\hbar\omega_{\rm LO} = 36$ meV in GaAs), then no single phonon can carry away all the electronic energy. Multiphonon decay processes are still allowed but the rate of these is negligible (except in certain narrow energy bands [13][2]). The dominant decay mechanism in dots can therefore be photon emission with a consequent enhancement of overall efficiency.

The size of each of the coupled dots is strongly constrained by the requirement that the energy difference between the excited and ground states exceeds the optic-phonon energy $\hbar\omega_{\rm LO}$. Specifically, the energy difference between the two lowest states of one of the dots in isolation must exceed $\hbar\omega_{\rm LO}$. The resulting maximum dot size L can be estimated from the energy spacing in a square well of size L

$$\frac{3\pi^2\hbar^2}{2m^*L^2} > \hbar\omega_{\rm LO}. \tag{1}$$

For GaAs, with an effective mass $m^* = 0.067m$, this implies dots smaller than $L \simeq 20$ nm in all three dimensions. Fig. 2(a) shows a schematic array of pairs of such coupled dots sandwiched between conducting sheets. The necessary size scales are close to current technology: dot arrays involving single quantum dots have been fabricated by electron-beam lithography with dot diameters of 57 nm [9], and arrays with dot diameters of 25 nm have been achieved via self-assembled growth [10], [11].

More is required than just small dots, however, since a laser also requires gain. Laser action will only occur if the gain coefficient $\gamma(\omega)$ exceeds the distributed loss

$$\gamma(\omega) > \alpha_I + \alpha_M, \tag{2}$$

where α_I is the bulk loss and $\alpha_M = (1/l)\log(1/R)$ is the loss through the mirrors. Equation (2) jointly constrains the minimum density of dot pairs and the uniformity of dot sizes. The gain is proportional to the three-dimensional (3-D) density of coupled dots N [14],

$$\gamma(\omega) = fN\sigma(\omega) \tag{3}$$

where f is the fraction of coupled dots with an electron in the excited state (we neglect the small fraction of dots with an electron in the ground state) and $\sigma(\omega)$ is the cross section. It is convenient to write $\sigma(\omega)$ as the product of an oscillator strength S and a normalized lineshape function $g(\omega)$:

$$\sigma(\omega) = Sg(\omega). \tag{4}$$

[2]The multiphonon rate can be significant in a narrow band around the optic phonon energy. The rate can also be significant at low multiples of the optic-phonon energy, with the total rate falling off as $(0.04)^N$ for N-optic-photon emission in GaAs. However, these resonances can be avoided by proper tuning of the energy difference between the excited and ground states.

In the dipole approximation, the oscillator strength is given by [15]

$$\begin{aligned} S &= \frac{4\pi^2\alpha\omega_{fi}}{n}|\langle f|\boldsymbol{r}\cdot\hat{\boldsymbol{e}}|i\rangle|^2 \\ &\simeq \frac{4\pi^2\alpha\omega_{fi}}{n}\left(\frac{td}{\hbar\omega_{fi}}\right)^2 \end{aligned} \tag{5}$$

where $\alpha = e^2/\hbar c \simeq 1/137$ is the fine structure constant, n is the index of refraction, and ω_{fi} is the transition frequency between initial and final states. The dipole matrix element between initial and final states $\langle f|\boldsymbol{r}\cdot\hat{\boldsymbol{e}}|i\rangle$ projects the polarization direction $\hat{\boldsymbol{e}}$ on the dipole moment. In the coupled dots, the transition dipole moment lies purely along the current direction so the radiation will be polarized in that direction. In the second line of (5), we have approximated the dipole matrix element by the product of the distance between the dots f and interdot hybridization $t/\hbar\omega_{fi}$, where t is the tunnel coupling between dots.[3] The remaining factor in the cross section is the normalized lineshape function $g(\omega)$. It is realistic to assume that inhomogeneous broadening due to disorder will determine the lineshape. Assuming the transition frequency is Gaussian distributed, with standard deviation $\Delta\omega$ due to defects and geometrical variations in the quantum dots, one finds a peak gain coefficient of [16][4]

$$\gamma_{\rm peak} = \frac{fNS}{\sqrt{2\pi}\Delta\omega}. \tag{6}$$

By equating the peak gain in (6) to the total loss, and employing a constraint on the interdot hybridization $t/\hbar\omega_{fi}$, we can state the joint requirement on density and uniformity for a functional quantum-dot laser. The distributed loss for a semiconductor injection laser is at least 10 cm^{-1} [14]. The interdot hybridization $t/\hbar\omega_{fi}$ must be sufficiently small that the spontaneous emission rate $w_{\rm sp}$ dominates the leakage rate from the excited state through the downstream barrier. In turn, the spontaneous emission rate must be smaller than the escape rate from the ground state. Assuming a fixed escape rate Γ through the downstream barrier, these inequalities imply[5]

$$(t/\hbar\omega_{fi})^2\Gamma < w_{\rm sp} < \Gamma \tag{7}$$

which clearly limits the hybridization to $(t/\hbar\omega_{fi})^2 \lesssim 1/10$. (However, this condition can be relaxed by additional band-structure engineering [12]. Further, assuming a transition energy of 100 meV, interdot spacing of $d = 10$ nm, and index of refraction $n = 3$, we find an excited coupled-dot density to broadening energy ratio of

$$\frac{fN}{\hbar\Delta\omega} \simeq 2.4 \times 10^{16} \text{ cm}^{-3} \text{ eV}^{-1}. \tag{8}$$

Hence, a 10% disorder broadening of the transition energy, $2\Delta\omega = 0.1\omega_{fi}$, corresponding to 5% nonuniformity, and an

[3]The interdot hybridization $t/\hbar\omega_{fi}$ gives the ratio of amplitudes in the dots, i.e., a hybridization of $\sqrt{1/10}$ implies a probability of $(\sqrt{1/10})^2 = 1/10$ of finding the first excited state electron in the downstream dot.

[4]An equivalent expression is employed to analyze the performance of the QW cascade laser.

[5]It is worth noting that since $w_{\rm sp} \sim t^2$, the first inequality in (7) provides a condition on the escape rate through the downstream barrier, $\Gamma < 4\alpha n\omega_{fi}^3 d^2/(3c^2)$.

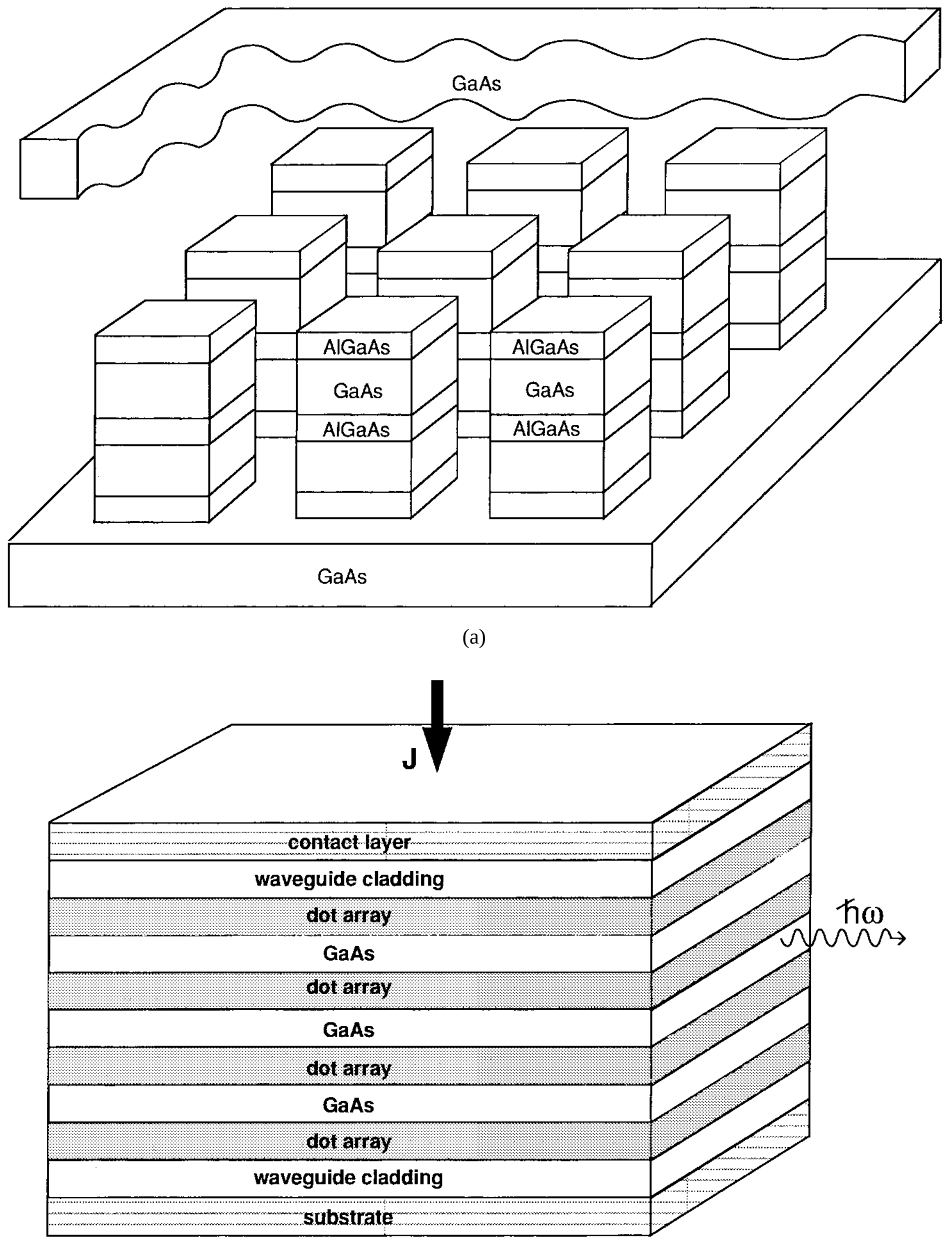

Fig. 2. (a) Schematic representation of coupled-quantum-dot array. For laser operation, the space between pairs of dots (shown as pillars) must be insulating so that a vertically directed current is constrained to flow through the dots. The conducting regions immediately above and below each pair of dots may be connected to form a continuous sheet as shown. (b) Stacked layers of coupled-quantum-dot arrays, in the cascade configuration developed for the QW laser by Faist *et al.* [1].

excited fraction near $f = 1$, implies a minimum density of one coupled dot pair per (200 nm)^3 volume.

To achieve this average density of coupled dots throughout the region occupied by the lasing mode requires a true 3-D structure. One can envision a layered structure, each layer consisting of a dense array of coupled quantum dots, as sketched in Fig. 2(b), with an overall density satisfying the conditions for gain. The stacking of arrays of quantum dots is analogous to the cascade of coupled QW's employed by Faist *et al.* [1], so the resulting device should properly be called a "quantum-dot cascade laser."

Finally, we can estimate the threshold current for such a device. Since single-phonon decay processes have been eliminated, the current flowing through each pair of dots need only be adequate to replenish losses due to spontaneous emission, multiphonon decays, and leakage. The leakage current can be controlled via suitable band structure engineering [12] and should be smaller than the spontaneous emission rate (7). Multiphonon-assisted recombination is exponentially suppressed [13], so the ideal threshold current should be determined by the total spontaneous emission rate [15]

$$w_{\rm sp} = \frac{4\alpha n\omega_{fi}^3}{3c^2}|\langle f|\boldsymbol{r}|i\rangle|^2 \simeq \frac{4\alpha n\omega_{fi}}{3}\left(\frac{td}{\hbar c}\right)^2. \tag{9}$$

Using the same parameters as above, one finds a threshold current of $J_t = ew_{\rm sp} \simeq 1.6$ pA per coupled dot pair. For a uniform array of dots in three dimensions, this gives a current density of 4.9 mA/cm^2. While it is not fair to compare the calculated performance of a proposed device to the actual performance of a real device, it is still striking that this threshold current density is some six-and-a-half orders of magnitude lower than the QW cascade laser value of 14 kA/cm^2 [1].

In addition to the low threshold current, the quantum-dot cascade laser offers several other advantages. The operation should be essentially temperature-independent provided kT is smaller than the level spacing in the dots. Since the levels are discrete, there is no thermal broadening, and since phonon decay processes are eliminated there is no increase in the decay rate with increasing phonon occupation. Finally, the operation frequency is in principle tunable by the applied bias, as in the QW structure [17].

It is important to note that the requirement of a pair of quantum dots in this proposal is solely to prevent leakage via tunneling from the excited state. If this tunneling can be prevented by bandstructure engineering, e.g., by placement of a superlattice with a forbidden band at the excited-state energy, then the lasing transition can take place within a single dot. Such "vertical-transition" schemes have been used successfully in the QW cascade laser [18].

In conclusion, we have proposed and analyzed a version of the quantum cascade laser [1] based on quantum dots rather than QW's. The quantum dot version offers the possibility of a several-order-of-magnitude reduction in the threshold current by eliminating single phonon decays. The constraints on dot size (10–20 nm), and dot density and uniformity [one coupled dot pair per (200 nm)^3 with 5% nonuniformity] are close to current technology. We hope to have stimulated interest in constructing a low-threshold semiconductor laser based on electronic transitions in quantum dots.

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