Kondo Impurity in a Mesoscopic Ring: Charge Persistent Current

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We study the influence of a magnetic impurity or ultrasmall quantum dot on the charge persistent current of a mesoscopic ring. The system consists of electrons in a one-dimensional ring threaded by spin–dependent Aharonov-Bohm/Casher fluxes, coupled via an antiferromagnetic exchange interaction to a localized electron. By passing to a basis of electron states with definite parities, the problem is mapped onto a Kondo model for the even-parity channel plus free electrons in the odd-parity channel. The twisted boundary conditions representing the fluxes couple states of opposite parity unless the twist angles satisfy $\phi_\alpha = f_\alpha \pi$, where f_α are integers, with spin index $\alpha =\uparrow,\downarrow$. For these special values of ϕ_α, the model is solved exactly by a Bethe ansatz. Special cases are investigated in detail. In particular we show that the charge stiffness in the case $\phi_\uparrow = \phi_\downarrow$ is insensitive to the presence of the magnetic impurity/quantum dot.

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1. INTRODUCTION

Recent experimental breakthroughs in identifying a Kondo effect in a quantum dot connected to leads have led to a flurry of activities in *mesoscopic Kondo physics.*[1] In a parallel development, data on Kondo scattering from a *single* magnetic impurity has also been reported.[2] With Kondo physics now available in the laboratory at the mesoscopic scale, the full richness of this paradigm of many-body physics can be explored.

One of the basic fingerprints of coherent electron transport is that of the Aharonov-Bohm (AB) effect, which has already been observed in conducting microstructured rings coupled to a quantum dot.[3] A natural question to ask is how the AB effect and its dual, the Aharanov-Casher (AC) effect, in a ring coupled to a quantum dot and hence e.g. the persistent currents would be modified by the many-body correlations present in the Kondo regime.

As is well-known, charge and spin persistent currents are the equilibrium responses of a multiply-connected system to a magnetic AB flux and/or an AC flux of a charged string (ABC fluxes) piercing the system.[4–7] The persistent current of a ring coupled via tunneling to a quantum dot was investigated via perturbation theory and numerical diagonalization by Büttiker and Stafford.[7] In this article, we study a variant of the problem where electrons in a one-dimensional (1D) ring threaded by spin-dependent ABC fluxes ϕ_α ($\alpha =\uparrow,\downarrow$) are coupled via antiferromagnetic exchange to a localized electron, representing a magnetic impurity or quantum dot. A detailed analysis shows that this model can be mapped onto the integrable Kondo model for special values of ϕ_α, corresponding to periodic and antiperiodic boundary conditions. The model is solved via Bethe ansatz for these special values of ϕ_α, and it is shown that the charge stiffness for $\phi_\uparrow = \phi_\downarrow$ is insensitive to the Kondo scattering, showing that spin-charge separation holds even on the mesoscopic scale in this model. However, for $\phi_\uparrow \neq \phi_\downarrow$, the charge persistent current is affected by the presence of the magnetic impurity, as can be shown by a detailed analysis of the Bethe ansatz equations.

2. EXACTLY SOLVABLE MODEL

The system we are considering is described in the continuum limit by the 1D Hamiltonian,

$$H = -\frac{\hbar^2}{2m}\sum_\alpha \int_0^L dx \psi_\alpha^\dagger(x)\partial_x^2\psi_\alpha(x) + \lambda\sum_{\alpha,\beta}\psi_\alpha^\dagger(0)\vec{\sigma}_{\alpha\beta}\psi_\beta(0)\cdot\vec{S}, \qquad (1)$$

where $\lambda > 0$ is an antiferromagnetic Kondo coupling, m is the electron mass, L is the circumference of the ring, $\vec{S}$ is the impurity spin (located at $x = 0$), and ψ_α is an electron field with spin index $\alpha =\uparrow,\downarrow$. The effect of the fluxes ϕ_α has been gauged away[4] and encoded in twisted boundary conditions:

$$\psi_\alpha(L) = e^{i\phi_\alpha}\psi_\alpha(0), \qquad (2)$$

where $\phi_{\uparrow,\downarrow} = 2\pi(\Phi/\Phi_0 \pm 4\pi\tau/F_0)$. Here Φ is the magnetic flux enclosed by the ring and τ the line charge density of a charged string passing through the center of the ring. $\Phi_0 = hc/e$ is the elementary magnetic flux quantum, with $F_0 = hc/\mu$ its electric analogue, μ being the projection of the magnetic moment along the string.

We are interested in an exact solution of the problem described by (1) and (2), with a particular eye on how the Kondo interaction in (1) may affect the persistent current induced by the boundary phase angles ϕ_α describing the ABC fluxes. Since the essential physics of the system is confined to a small region around the left and right Fermi points, we can linearize the quadratic dispersion in (1) around $\pm k_{\mathrm{F}}$ and introduce left (l) and right (r) moving chiral fields:

$$\psi_\alpha(x) \sim \mathrm{e}^{-ik_{\mathrm{F}}x}\psi_{l,\alpha}(x) + \mathrm{e}^{ik_{\mathrm{F}}x}\psi_{r,\alpha}(x). \tag{3}$$

The Hamiltonian then becomes:

$$H = H_0 + H_{\mathrm{imp}}, \tag{4}$$

with

$$H_0 = \frac{v_{\mathrm{F}}}{2\pi}\sum_\alpha \int_0^L dx\, \left(\psi^\dagger_{l,\alpha}(x) i\partial_x \psi_{l,\alpha}(x) - \psi^\dagger_{r,\alpha}(x) i\partial_x \psi_{r,\alpha}(x)\right), \tag{5}$$

and

$$H_{\mathrm{imp}} = \lambda \sum_{\alpha,\beta} \left(\psi^\dagger_{l,\alpha}(0) + \psi^\dagger_{r,\alpha}(0)\right) \vec{\sigma}_{\alpha\beta} \left(\psi_{l,\beta}(0) + \psi_{r,\beta}(0)\right) \cdot \vec{S}. \tag{6}$$

To make progress, it is convenient to pass to a basis of definite parity fields (*Weyl basis*):

$$\psi_{\mathrm{even},\alpha}(x) = \frac{1}{\sqrt{2}}\left(\psi_{r,\alpha}(x) + \psi_{l,\alpha}(-x)\right), \tag{7}$$

an even–parity, right–moving electron field, and

$$\psi_{\mathrm{odd},\alpha}(x) = \frac{1}{\sqrt{2}}\left(\psi_{r,\alpha}(-x) - \psi_{l,\alpha}(x)\right), \tag{8}$$

an odd–parity, left–moving field. One should note that the assignment of chirality (*left/right*) to parity (*odd/even*) is not intrinsic, but a property of the particular transformations (7) and (8). This is analogous to a gauge–fixing condition. In this basis the Hamiltonian takes the form:

$$H = H_0^{\mathrm{odd}} + H_0^{\mathrm{even}} + H_{\mathrm{imp}}^{\mathrm{even}}, \tag{9}$$

where

$$H_0^{\mathrm{even}} = -\frac{v_{\mathrm{F}}}{2\pi}\sum_\alpha \int_0^L dx \psi^\dagger_{\mathrm{even},\alpha}(x) i\partial_x \psi_{\mathrm{even},\alpha}(x) \tag{10}$$

and

$$H_0^{\mathrm{odd}} = \frac{v_{\mathrm{F}}}{2\pi}\sum_{\alpha}\int_0^L dx\psi^{\dagger}_{\mathrm{odd},\alpha}(x)i\partial_x\psi_{\mathrm{odd},\alpha}(x) \tag{11}$$

describe independent relativistic electrons, and the impurity contribution is now also diagonal:

$$H_{\mathrm{imp}}^{\mathrm{even}} = \lambda\sum_{\alpha,\beta}\psi^{\dagger}_{\mathrm{even},\alpha}(0)\vec{\sigma}_{\alpha\beta}\psi_{\mathrm{even},\beta}(0)\cdot\vec{S}. \tag{12}$$

We recognize $H_K^{\mathrm{even}} \equiv H_0^{\mathrm{even}} + H_{\mathrm{imp}}^{\mathrm{even}}$ as the chiral Hamiltonian of the spin-S Kondo model.

While the even and odd parity channels are decoupled in the Hamiltonian, they become connected by the twisted boundary conditions (2):

$$\begin{pmatrix} \psi_{\mathrm{even},\alpha}(L) \\ \\ \psi_{\mathrm{odd},\alpha}(L) \end{pmatrix} = \begin{pmatrix} \cos\phi_\alpha & i\sin\phi_\alpha \\ \\ -i\sin\phi_\alpha & \cos\phi_\alpha \end{pmatrix}\begin{pmatrix} \psi_{\mathrm{even},\alpha}(0) \\ \\ \psi_{\mathrm{odd},\alpha}(0) \end{pmatrix}, \tag{13}$$

where in (3) we have taken $k_F = (2\pi/L)n$, with n an integer. However, for the special values $\phi_\alpha = f_\alpha\pi$, where f_α is an integer, the matrix in Eq. (13) reduces to a multiple of the unit matrix, and the even and odd parity states decouple from each other entirely. One can then solve H_K^{even} by the Bethe ansatz.[8] Thus, our original problem in (1) and (2) has collapsed to an exactly solvable problem for $f_\alpha \in Z$, consisting of a left-moving odd-parity branch of independent relativistic electrons, together with a (decoupled) right-moving even-parity branch defined by the 1D Kondo model. For generic values of ϕ_α, it is not possible to choose a basis which renders the Hamiltonian and the boundary conditions simultaneously diagonal, strongly suggesting that the model is not integrable in general. This is in apparent contradiction to recent claims in the literature[11] about the integrability of the related Anderson ring threaded by an Aharonov-Bohm flux of arbitrary strength.

From Eq. (12), the impurity is seen to couple only to the spin current of the electrons, suggesting, via the dynamic spin-charge separation in 1D, that the charge persistent current is insensitive to the presence of the impurity. Although this indeed turns out to be the case—as we shall confirm via a Bethe ansatz analysis—some caveats are appropriate at this point: First, the persistent current is a boundary effect and, as such, could be influenced by *non-dynamical* selection rules for combining charge and spin.[9,10] Secondly, and possibly reflecting this, a magnetic impurity *does* affect the charge current of a *chiral* ring of free electrons (with all electrons moving in the same direction).[12] In any event, it is instructive to study the exact mechanism by which the charge persistent current in the present problem avoids any influence from the impurity. Moreover, the above is only true - as our exact Bethe ansatz solution shows - for $\phi_\uparrow = \phi_\downarrow$. In general, there is a marked effect of the presence of the magnetic impurity on the charge persistent current.

To carry out this analysis, we first need to consider how to properly define a persistent current for relativistic electrons, i.e. for electrons with a *linear* dispersion.

3. CHARGE PERSISTENT CURRENT FOR RELATIVISTIC ELECTRONS

In the usual treatment of independent 1D electrons,[5,6] the persistent current is obtained by summing the partial currents $I_n = -(e/\hbar)\partial E_n/\partial\phi$ over all occupied levels n. This approach clearly fails for the relativistic electrons in (10) and (11), since the corresponding linear dispersions

$$E_r = \hbar v_{\mathrm{F}} \frac{2\pi n_r + \phi}{L}, \qquad n_r = 0, 1, 2, ..., n_F \tag{14}$$

and

$$E_l = \hbar v_{\mathrm{F}} \frac{-2\pi n_l + \phi}{L}, \qquad n_l = 1, 2, ..., n_F \tag{15}$$

imply that $\partial E_n/\partial\phi = \text{const.}$ for all levels n. (Here, for simplicity, we consider a system of spinless electrons in which the total number of electrons $2n_F + 1$ is odd.) To recover the known results for the persistent current, we must thus use an alternative approach. Let us introduce *flux-dependent* particle numbers

$$N_{r/l}(\phi) = \frac{L}{2\pi}[|k_{r/l,F}(\phi)| - |k_{r/l,F}(0)|], \tag{16}$$

where $k_{r/l,F}$ are flux-dependent Fermi momenta, associated with the highest occupied level on the respective branch. The Fermi momenta $k_{r,F}$ and $k_{l,F}$ are cutoff dependent, and need not be equal, since only the right-movers couple to the impurity in Eq. (9). However, provided the cutoffs are chosen independent of ϕ, $N_{r/l}(\phi)$ are insensitive to the cutoffs, and describe the physical response of the system to an ABC flux. The persistent current is then

$$I(\phi) = -\frac{ev_F}{L}[N_r(\phi) - N_l(\phi)]. \tag{17}$$

It should be pointed out that the charge velocity v_F is in general subject to renormalization due to electron-electron interactions.

With the choice of representative levels in (14) and (15) (note in particular that the zero mode of the original problem (1) is assigned to *one* branch only) it is easy to verify that (16) and (17) exactly reproduce the known result for the persistent current of an odd number of spinless 1D electrons.[6] Our construction, introduced here *ad hoc*, can trivially be extended to spinful particles and put on a firm basis by a proper analysis of the cutoff procedure for 1D relativistic electrons in the presence of an Aharonov-Bohm flux.[13] In short, a flux-dependent particle number as in (16) is the trade-off that guarantees that physical observables remain independent of the choice of cutoff which bounds the spectrum of a finite system from below.

Given (16) and (17), the problem is now reduced to calculating how the Fermi momenta $k_{r/l,\mathrm{F}}$ depend on the flux and the coupling of the electrons to the magnetic impurity. For this, we turn to a finite-size Bethe ansatz analysis.

4. FINITE–SIZE BETHE ANSATZ

To obtain the Fermi momenta $k_{r/l,F}(\phi)$ for a finite ring, we apply the techniques of the *Bethe ansatz* for finite systems, developed previously for the 1D Hubbard model.[14] As pointed out above, our model is only integrable for $\phi_\alpha = f_\alpha \pi$, with f_α an integer. For $f_\alpha \in Z$, the nested Bethe ansatz equations which diagonalize H are

$$Lk_{n_l} = -2\pi n_l + f_c \pi + \left(\frac{2M_{\mathrm{odd}}}{N_{\mathrm{odd}}} - 1\right) f_s \pi + \frac{2\pi}{N_{\mathrm{odd}}} \sum_{\delta=1}^{M_{\mathrm{odd}}} J_\delta, \tag{18}$$

$$Lk_{n_r} = 2\pi n_r + f_\downarrow \pi + \sum_{\gamma=1}^{M_{\mathrm{even}}} \left[\Theta(2\Lambda_\gamma - 2) - \pi\right], \tag{19}$$

$$N_{\mathrm{even}}\Theta(2\Lambda_\gamma - 2) + \Theta(2\Lambda_\gamma) = 2\pi I_\gamma + (f_\uparrow - f_\downarrow)\pi + \sum_{\delta=1}^{M_{\mathrm{even}}} \Theta(\Lambda_\gamma - \Lambda_\delta), \tag{20}$$

where k_{n_l} are the pseudomomenta characterizing the N_{odd} odd-parity left movers which decouple from the impurity, M_{odd} of which have spin down, and k_{n_r} are pseudomomenta characterizing the N_{even} even-parity right movers, M_{even} of which have spin down, and $\{\Lambda_\gamma, \gamma = 1, \cdots, M_{\mathrm{even}}\}$ are a set of auxiliary variables known as spin-rapidities. The scattering phase shifts are given by $\Theta(x) = 2\tan^{-1}(x/c)$, with $c = 2\lambda/(1 - 3\lambda^2/4)$. We have also defined $f_{c,s} = (f_\uparrow \pm f_\downarrow)/2$. Eq. (18) simply gives the quantum numbers of free, chiral electrons, written in the Bethe ansatz basis. The Bethe ansatz equation (19) describes the charge degrees of freedom in the even channel (holons), while Eq. (20) describes the spin degrees of freedom in the even channel (spinons). Eqs. (19) and (20) differ from the Bethe ansatz equations derived previously for the Kondo model[8] only by the addition of the ABC fluxes $\phi_\alpha = f_\alpha \pi$.

Let us consider for the moment the case of a spin-independent flux $\phi_\uparrow = \phi_\downarrow = \phi$, corresponding to the case where only a magnetic flux threads the ring and there is no charged string passing through the ring (AB flux only). The persistent current is an odd function of ϕ by symmetry,[4] and is analytic, except at values of ϕ corresponding to level crossings. We are interested in the persistent current for small values of the AB flux. Choosing the total numbers of both up- and down-spin electrons to be odd excludes a level crossing at $\phi = 0$. The leading mesoscopic behavior of the persistent current is then

$$I(\phi) = -D_c \phi / L + \mathcal{O}(\phi^3/L^3), \tag{21}$$

where D_c is the charge stiffness. Eq. (21) holds on general grounds independent of whether the model is integrable or not.

The choice of quantum numbers $\{n_l, J_\delta, n_r, I_\gamma\}$ specifies the quantum state of the system. Generically, there are one or more level crossings[15] between $f = 0$ and $f = 1$. To determine the charge stiffness, however, we only need to consider the state which evolves adiabatically from the ground state at $f = 0$ as ϕ is increased. This state is given by $M_{\text{even/odd}} = (N_{\text{even/odd}} +/-1)/2$, (with $N_{\text{even/odd}}$ odd for simplicity), with integer-spaced quantum numbers $\{n_l, J_\delta, n_r, I_\gamma\}$ in the symmetric ranges $-(N_{\text{odd}} - 1)/2 \le n_l \le (N_{\text{odd}} - 1)/2$, $-(M_{\text{odd}} - 1)/2 \le J_\delta \le (M_{\text{odd}} - 1)/2$, $-(N_{\text{even}} - 1)/2 \le n_r \le (N_{\text{even}} - 1)/2$, and $-(M_{\text{even}} - 1)/2 \le I_\gamma \le (M_{\text{even}} - 1)/2$. The quantum numbers of the even-parity sector are the same as those of the Kondo model with periodic boundary conditions.[8]

Given a set of spin rapidities Λ_γ satisfying Eq. (20), we may calculate the sum in Eq. (19), and thus the momenta k_{n_r} are determined. One sees immediately that the total scattering phase shift of the dressed magnetic impurity is *independent* of f, so that $N_r = -N_l = f/2$. In addition, the charge velocity v_F is unrenormalized by interactions in this model.[8] The charge stiffness may be evaluated from Eqs. (17) and (21) as a finite difference $D_c = -LI(f = 1)/\pi = ev_F/\pi + \mathcal{O}(L^{-2})$, and is unaffected by the Kondo scattering. The persistent current for small Φ is thus

$$I = -\frac{ev_F}{L}\frac{2\Phi}{\Phi_0}, \tag{22}$$

which is identical to the result for free electrons. Eq. (22) indicates that spin-charge separation holds even at the mesoscopic scale in this model.

The analysis of the Bethe ansatz equations (18-20) in the case of general spin–dependent fluxes (however, still satisfying $\phi_\alpha = f_\alpha \pi$ with f_α an integer) is more involved and will be presented elsewhere.[13] However, an analysis of the Bethe ansatz equations (18-20) in the limiting cases $c \to 0$ and $c \to \infty$ indicates that the charge persistent current is markedly affected by the magnetic impurity when spin-dependent fluxes are present. This is because the AC effect induces a charge persistent current if the numbers of up and down spin electrons are not equal. As c is increased from 0 to ∞, the impurity screens exactly one electron spin, so the effective numbers of mobile up and down spin electrons are not equal in general.

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